Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 13

Time: 3 Hours. **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

Section A

Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

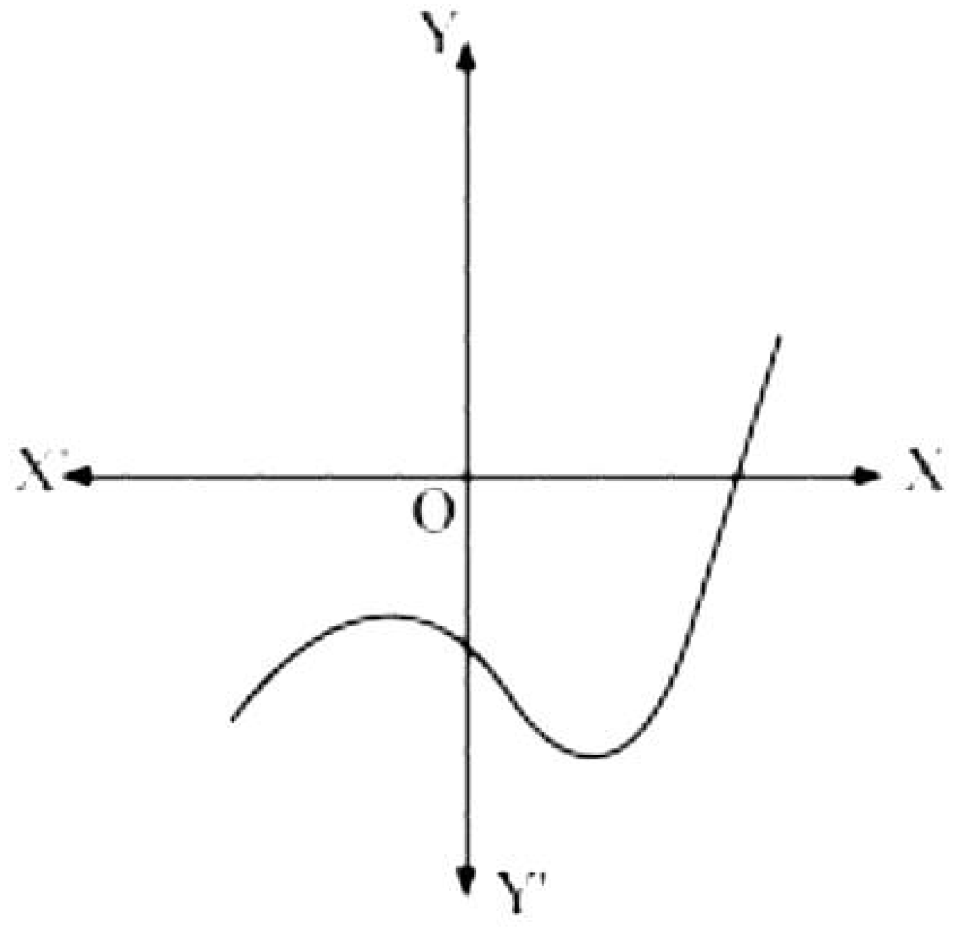
1. Find LCM of 336 and 54.

A. 3042
B. 3024
C. 3204
D. 3044

2. Find the discriminant of the following equation: $3x^2 - 2x + 8 = 0$

A. −94
B. 94
C. 92
D. −92

3. The graph of $y = p(x)$ is given in the following figure for some polynomial $p(x)$. Find the number of zeroes of $p(x)$.

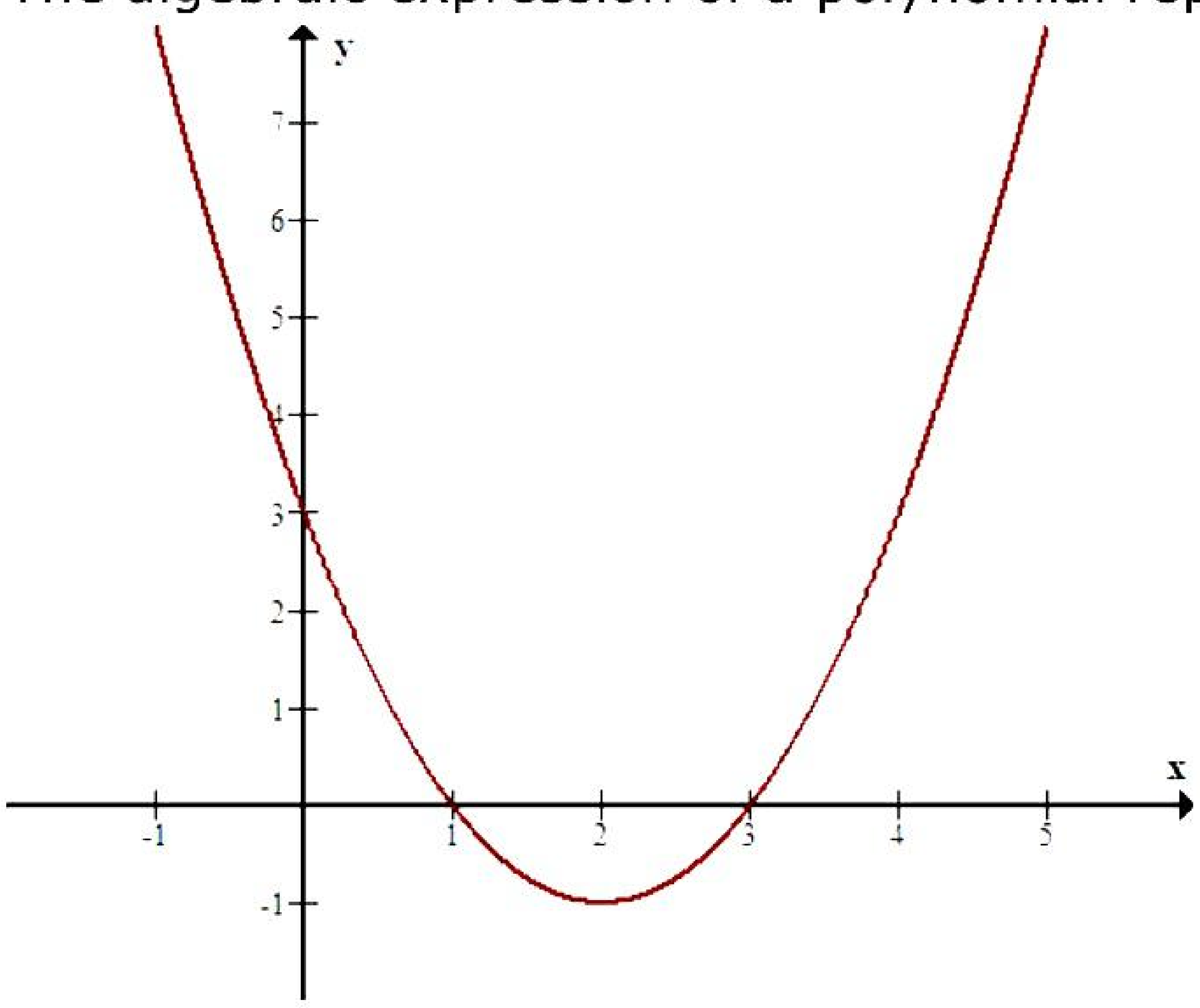


A. 2
B. 1
C. 0
D. 3

4. The sum of two numbers is 18, and they are alternate even numbers, find the largest out of the two.

A. 4
B. 6
C. 8
D. 10

5. The algebraic expression of a polynomial representing the following parabola is



A. $x^2 + 4x + 3$
B. $x^2 + 4x - 3$
C. $x^2 - 4x - 3$
D. $x^2 - 4x + 3$

6. Find the coordinates of the point equidistant from three points A(5, 3), B(5, −5) and C(1, −5).

A. (2, −1)
B. (3, −1)
C. (4, −1)
D. (5, −1)

7. If sum of the zeros of a cubic polynomial $ax^3 + (-7x^2) + (-13x) + (d)$ is $\frac{7}{5}$ and product of zeroes is 1, then the values of 'd' and 'a' are

A. 5, 5
B. −5, 5
C. 5, −5
D. −5, −5

8. The perimeters of two similar triangles ABC and PQR are 32 cm and 24 cm, respectively. If PQ = 12 cm, then find AB.

A. 24 cm
B. 32 cm
C. 12 cm
D. 16 cm

9. Following is not a test of similarity

A. SSS
B. SAS
C. AAA
D. SSA

10. If $\Delta ABC \sim \Delta DEF$ such that 2AB = DE and BC = 6 cm, find EF.

A. 10 cm
B. 12 cm
C. 1 cm
D. 4 cm

11. Find the value of θ if $\tan\theta = \cot\theta$

A. 30°
B. 60°
C. 45°
D. 90°

12. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of θ.

A. 100°
B. 70°
C. 90°
D. 80°

13. Raju and Ravi are standing at an equal distance from a Pole, but on the opposite sides. Both are looking at the top of the pole. The angle of elevation of Raju's eye sight from the horizontal is 30°, and that of Ravi is 45°.
If so, then which of the following statement is true?
A. Raju is taller than Ravi
B. Ravi is taller than Raju
C. Both Raju and Ravi have same heights
D. Data is insufficient to compare the heights of Raju and Ravi

14. Find the area of a sector with radius 7 cm and central angle 90°.
A. 38 cm^2
B. 39 cm^2
C. 38.5 cm^2
D. 37.5 cm^2

15. The total surface area of a right circular cylinder is given by
A. $2\pi r(r + h)$
B. $2\pi r(r - h)$
C. $2r(r + h)$
D. $\pi r(r + h)$

16. Find the modal class from the following table:

Size	Frequency
45-55	7
55-65	12
65-75	17
75-85	30
85-95	32
95-105	6
105-115	10

A. 75-85
B. 85-95
C. 95-105
D. 105-115

17. Cards bearing numbers 1, 3, 5,..., 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing a prime number less than 15.
A. 3/18
B. 4/18
C. 5/18
D. 7/18

18. Netra has a total of 361 songs in her playlist out of which 165 are Hindi, 87 are Punjabi and 109 are English. She starts listening to music by choosing the first song from Hindi category. She will continue listening if the next song which will be played automatically is Punjabi. What is the probability that Netra will continue listening to music?

A. 87/361
B. 87/360
C. 109/361
D. 109/360

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

19. **Statement A (Assertion)**: From a solid cylinder whose height is 8 cm and radius is 6 cm, a conical cavity of height 8 cm and of base radius 6 cm is hollowed out. Hence the volume of the remaining solid will be 192 cm^3.

Statement R (Reason): Volume of remaining solid = Volume of the cylinder - Volume of the cone removed

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

20. **Statement A (Assertion)**: $\frac{4}{5}$, a, 2 are three consecutive terms of an AP only if $a = \frac{7}{5}$.

Statement R (Reason): If p, q and r are in A.P then $q - p = r - q$.

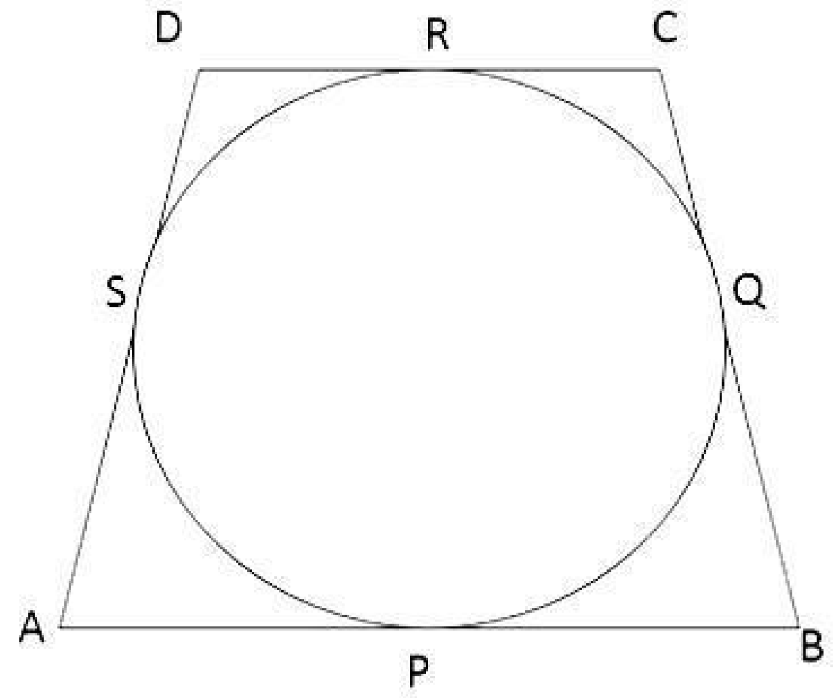
A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

Section B

21. Prove that $3 + 2\sqrt{5}$ is an irrational number. [2]

22. P and Q are the points on the sides AB and AC, respectively, of a $\triangle ABC$. If AP = 2 cm, PB = 4 cm, AQ = 3 cm and QC = 6 cm, show that BC = 3PQ. [2]

23. In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD. [2]



24. Prove that: $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$ [2]

OR

Prove that $\dfrac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta} = \dfrac{1 - \sin\theta}{\cos\theta}$

25. Find the area of a ring whose outer and inner radii are 23 cm and 12 cm. [2]

OR

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Section C

Section C consists of 6 questions of 3 marks each.

26. In a seminar, the number of participants in Hindi, English and Mathematics is 60, 84 and 108, respectively. Find the minimum number of rooms required, if in each room the same number of participants are to be seated and all of them being in the same subject. [3]

27. Find the zeroes of the quadratic polynomials $4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients. [3]

28. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train. [3]

OR

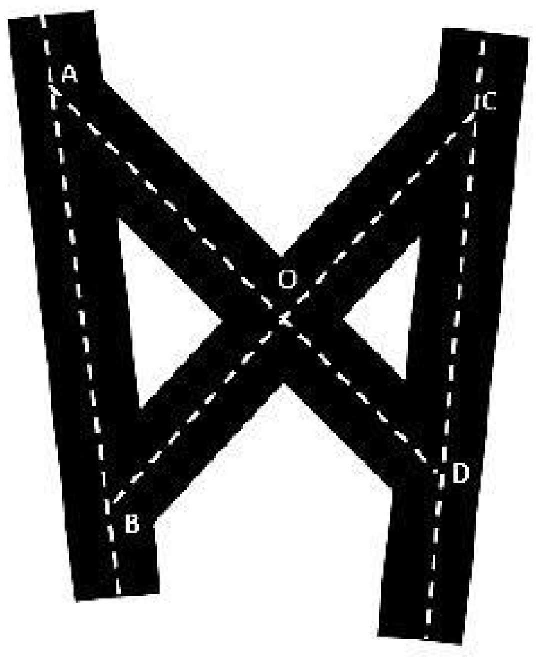
Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (x + y + 10)^\circ$, $\angle B = (y + 20)^\circ$, $\angle C = (x + y - 30)^\circ$ and $\angle D = (x + y)^\circ$.

29. Raja is walking away from the base of a lamp-post at a speed of 2 m/s. If the lamp is 3 m above the ground, find the length of his shadow after 4 seconds, if Raja's height is 90 cm. [3]

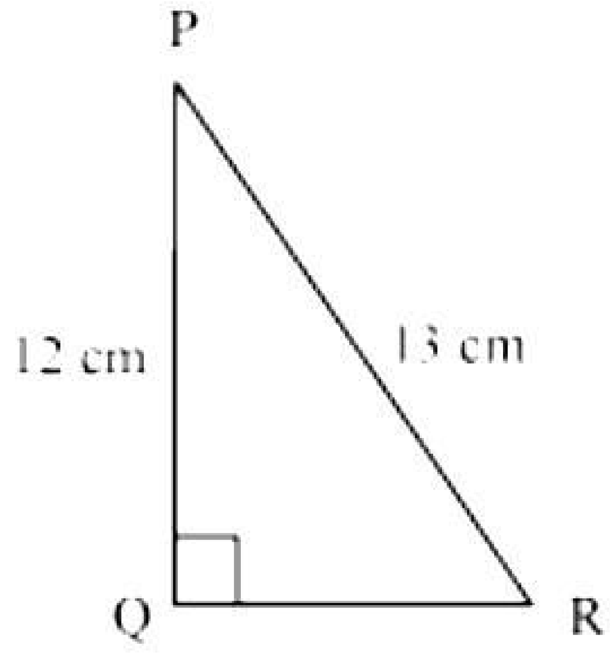
OR

We have a system of intersecting roads as shown in the figure.

If AO = 6 m, OB = 4 m, AB = 8 m, OD = 2 m and OC = 3 m, then find the length of CD.



30. In the given figure find tan P – cot R. [3]



31. Two dice are thrown simultaneously. What is the probability that [3]

i. 5 will not come up on either of them?

ii. 5 will not come up on at least one?

iii. 5 will come up at both dice?

Section D

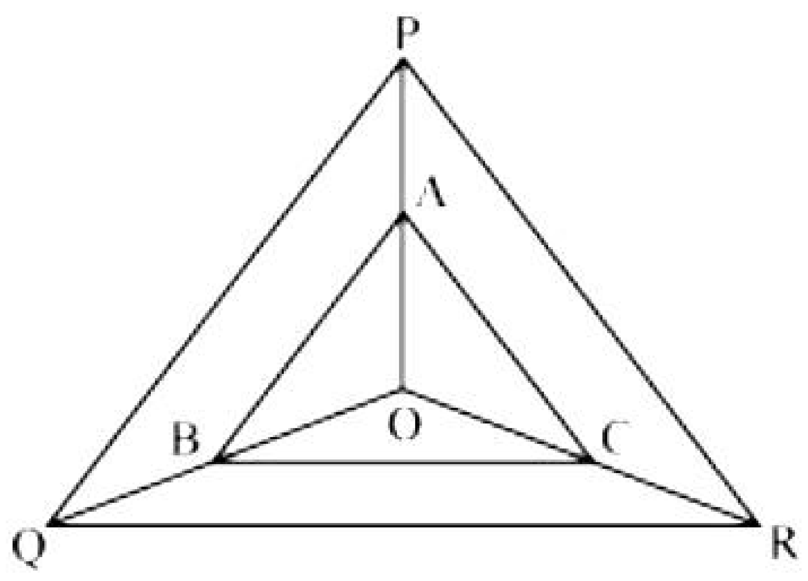
Section D consists of 4 questions of 5 marks each.

32. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age. [5]

OR

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

33. In figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR. [5]



34. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$. [5]

OR

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs. 500 per m^2.

35. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent. [5]

Number of days	0–6	6–10	10–14	14–20	20–28	28–38	38–40
Number of students	11	10	7	4	4	3	1

Section E

Case study based questions are compulsory.

36. Rukhsar is celebrating her birthday. She invited her friends. She bought a packet of chocolates which contains 120 chocolates. She arranges the chocolates such that in the first row there are 3 chocolates, in second there are 5 chocolates, in third there are 7 chocolates and so on.

i. Find the total number of rows of chocolates. [1]

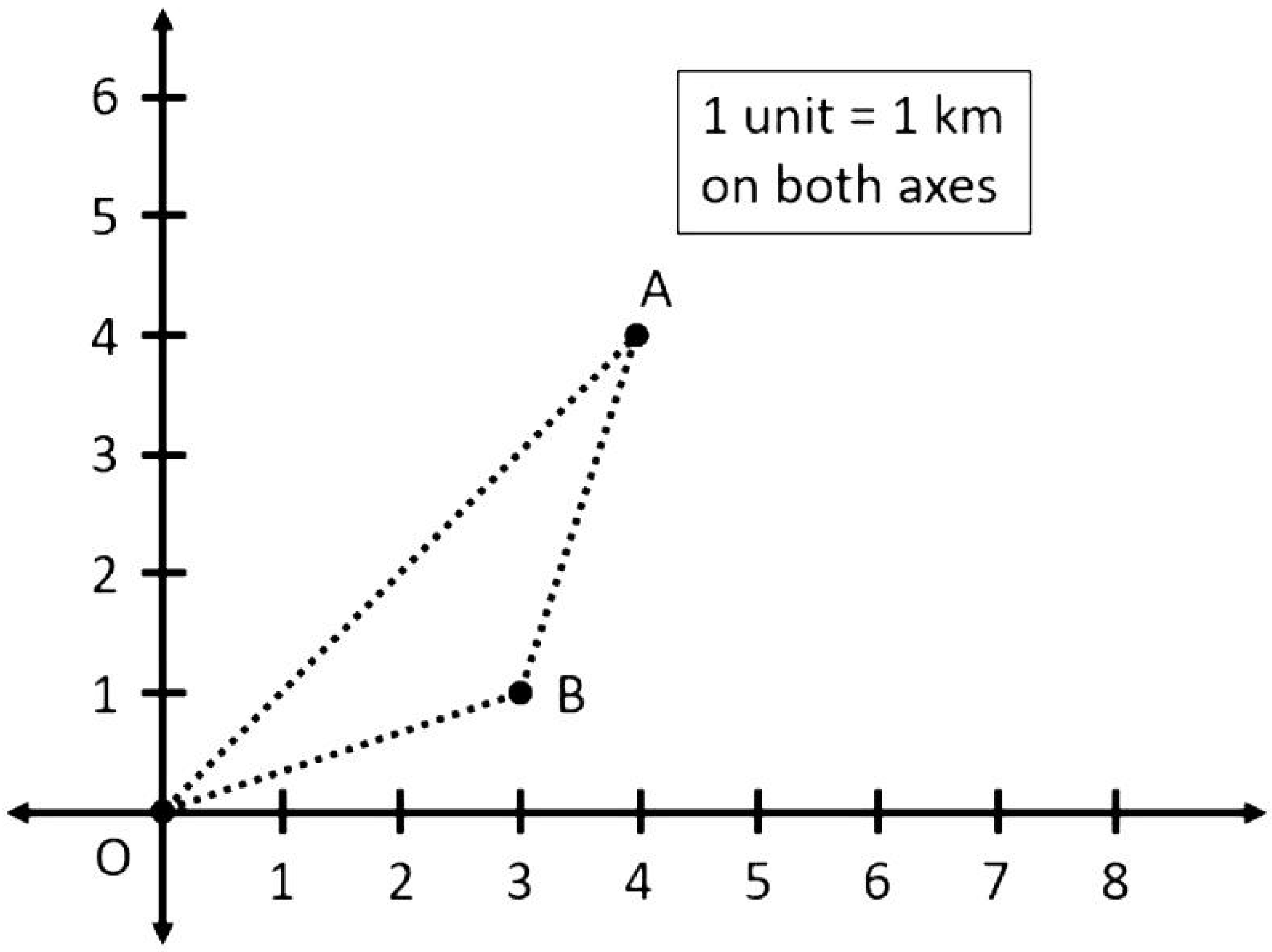
ii. How many chocolates are placed in last row? [2]

OR

Find the difference in number of chocolates placed in 7^{th} and 3^{rd} row. [2]

iii. If Rukhsar decides to make 15 rows, then how many total chocolates will be placed by her with the same arrangement? [1]

37. Bus number 735 travels from source O to A, and Bus number 736 travels from Source O to B, then reaches A. The routes taken by both the buses are shown below. Using the details given, answer the following questions.



i. Find the distance covered by Bus No. 735. [1]

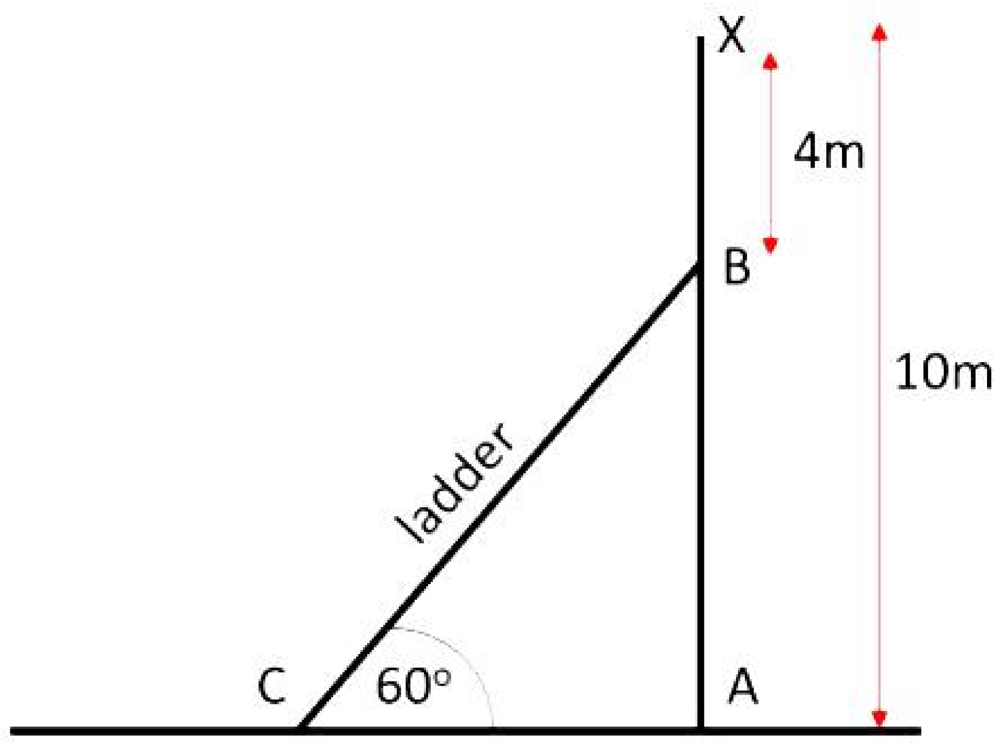
ii. Find the distance between locations B and A. [2]

OR

Find the distance between locations O and B. [2]

iii. Find the distance covered by Bus No. 736. [1]

38. Vinod, an electrician, has to repair an electric wire on the pole AX which is of height 10 m. For the repair, he needs to reach a point B which is 4 m below the top of the pole, using a ladder from the point C. The ladder makes an angle of 60° with the ground. Based on the above information, answer the following questions.



i. Find the length of AB. [1]

ii. If the ladder makes an angle of 60° with the ground, what is the distance between the foot of the ladder and the pole? [2]

OR

If the ladder makes an angle of 60° with the ground, then the length of ladder will be? [2]

iii. If AB = AC, what angle should the ladder make with the ground? [1]

Solution

Section A

1. Correct Option: B
Explanation:
Prime factorization of the numbers:
$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$
$54 = 2 \times 3 \times 3 \times 3$
LCM (336, 54)
$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7$
$= 3024$

2. Correct option: D
Explanation:
The given equation is $3x^2 - 2x + 8 = 0$
Comparing it with $ax^2 + bx + c = 0$, we get
$a = 3, b = -2, c = 8$
$\therefore D = \left(b^2 - 4ac\right) = \left[(-2)^2 - (4 \times 3 \times 8)\right]$
$= (4 - 96) = -92$

3. Correct option: B
Explanation:
The graph of p(x) intersects the x-axis at only 1 point.
So, the number of zeroes is 1.

4. Correct Option: D
Explanation:
Let the two numbers be x and y, hence
$x + y = 18$ ….. (i)
$x - y = 2$ ….. (ii) [Alternate even numbers have difference 2]
From (i) and (ii), we get
$2x = 20 \Rightarrow x = 10$
Substituting $x = 10$ in equation (i), we get
$10 + y = 18 \Rightarrow y = 18 - 10 = 8$

5. Correct Option: D
Explanation:
The parabola intersects X-axis at 1 and 3.
Therefore, 1 and 3 are the zeroes of polynomial representing given parabola.
Then, polynomial = x^2 – (sum of zeroes)x + product of zeroes
$= x^2 - (1 + 3)x + (1 \times 3)$
$= x^2 - 4x + 3$

6. Correct Option: B

Explanation:

Let the required point be P(x, y), then

PA = PB = PC

The points A, B, C are (5, 3), (5, −5) and (1, −5), respectively.

$\Rightarrow PA^2 = PB^2 = PC^2$

$\Rightarrow PA^2 = PB^2$ and $PB^2 = PC^2$

$PA^2 = PB^2$

$\Rightarrow (5-x)^2 + (3-y)^2 = (5-x)^2 + (-5-y)^2$

$\Rightarrow 25 + x^2 - 10x + 9 + y^2 - 6y = 25 + x^2 - 10x + 25 + y^2 + 10y$

$\Rightarrow -6y - 10y = 25 - 9$

$\Rightarrow -16y = 16$

$\Rightarrow y = -1$

and $PB^2 = PC^2$

$\Rightarrow (5-x)^2 + (-5-y)^2 = (1-x)^2 + (-5-y)^2$

$\Rightarrow 25 + x^2 - 10x + 25 + y^2 + 10y = 1 + x^2 - 2x + 25 + y^2 + 10y$

$\Rightarrow -10x + 2x = -24$

$\Rightarrow -8x = -24$

$\Rightarrow x = \frac{-24}{-8} = 3$

$\Rightarrow x = 3$

Hence, the point P is (3, −1).

7. Correct Option: B

Explanation:

Given cubic polynomial is $ax^3 + (-7x^2) + (-13x) + (d)$.

Now, Sum of the zeros $= \frac{7}{5} = -\frac{b}{a}$

$\Rightarrow \frac{7}{5} = -\frac{(-7)}{a}$

$\Rightarrow a = 5$

And, product of zeroes = 1

$\Rightarrow 1 = -\frac{d}{a} = -\frac{d}{5}$

$\Rightarrow d = -5$

8. Correct option: D

Explanation:

It is given that ΔABC and ΔPQR are similar triangles, so the corresponding sides of both triangles are proportional.

So, $\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{AB}{PQ}$

Let, $AB = x$ cm

Then, $\frac{x}{12} = \frac{32}{24} \Rightarrow x = \frac{32 \times 12}{24} = 16$ cm

Hence, AB = 16 cm.

9. Correct option: D

SSA is not a test of similarity, the angle should be included between the two sides.

10. Correct Option: B

Explanation:

$\Delta ABC \sim \Delta DEF$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{1}{2} = \frac{6}{EF}$

$\Rightarrow EF = 12$ cm

11. Correct option: C

Explanation:

Since, $\tan 45° = \cot 45°$

$\Rightarrow \theta = 45°$

12. Correct option: C

Explanation:

$2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow 2(1 - \cos^2\theta) - \cos^2\theta = 2$

$\Rightarrow 2 - 2\cos^2\theta - \cos^2\theta = 2$

$\Rightarrow 2 - 3\cos^2\theta = 2$

$\Rightarrow 3\cos^2\theta = 0$

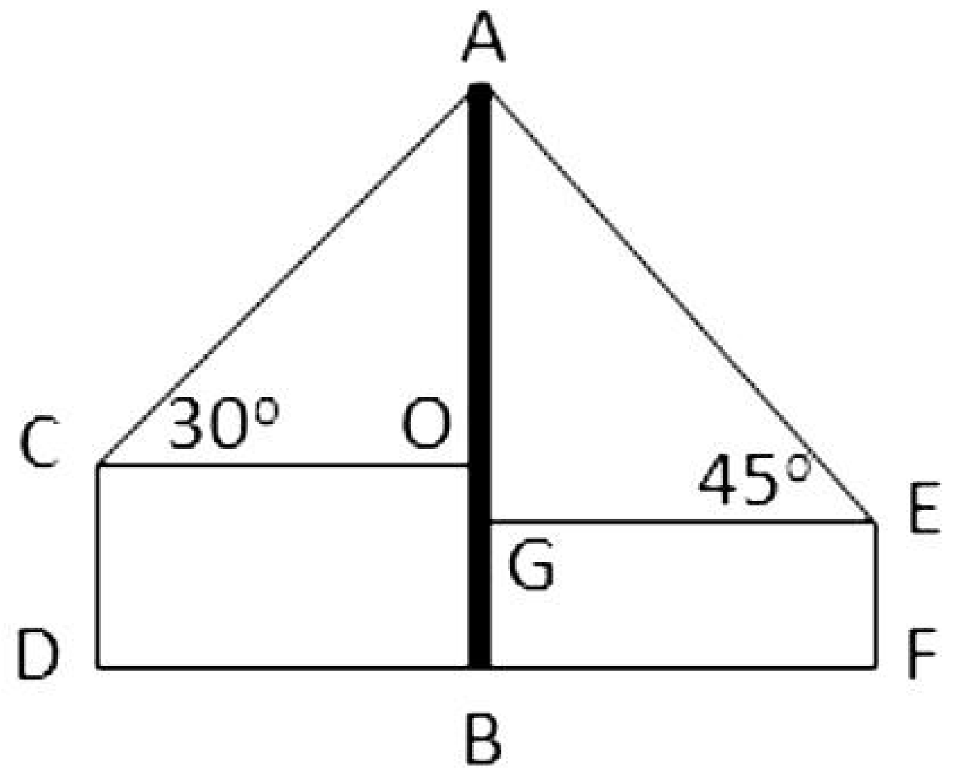
$\Rightarrow \cos^2\theta = 0$

$\Rightarrow \cos^2\theta = \cos^2 90°$

$\Rightarrow \theta = 90°$

13. Correct option: A

Explanation:



In the figure above,
CD represents the height of Raju.
EF represents the height of Ravi.
AB represents the height of pole.
Now, in respective triangles we have

$\tan 30^\circ = \frac{AO}{CO}$ and $\tan 45^\circ = \frac{AG}{GE}$

$\therefore \frac{1}{\sqrt{3}} = \frac{AO}{CO}$ and $1 = \frac{AG}{GE}$

Now, CO = GE

$\therefore \frac{1}{\sqrt{3}} = \frac{AO}{CO}$ and $1 = \frac{AG}{CO}$

$\therefore CO = \sqrt{3}AO$ and $CO = AG$

$\therefore AG = \sqrt{3}AO$

$\therefore AG > AO$

$\therefore AB - GB > AB - OB$

$\therefore -GB > -OB$

$\therefore GB < OB$

$\therefore OB > GB$

$\therefore CD > EF$

$\therefore$ Raju's height > Ravi's height

14. Correct Option: C

Explanation:

Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (7)^2$

$= 38.5 \text{ cm}^2$

15. Correct option: A

Explanation:

The total surface area of a right circular cylinder is given by $2\pi rh + 2\pi r^2$

$= 2\pi r(r + h)$

16. Correct option: B

Explanation:

As the class 85–95 has the maximum frequency, it is the modal class.

17. Correct option: C

Explanation:

There are 18 cards having numbers 1, 3, 5, ..., 35 kept in a bag.

Prime numbers less than 15 are 3, 5, 7, 11, 13.

There are 5 numbers.

$\therefore$ Probability that a card drawn bears a prime number less than 15 = $\frac{5}{18}$

18. Correct option: B

Explanation:

One Hindi song is already played.

So, there are 360 songs left from which 1 song will be played automatically.

Total outcomes = 360

There are only 87 Punjabi songs.

Favorable outcomes = 87

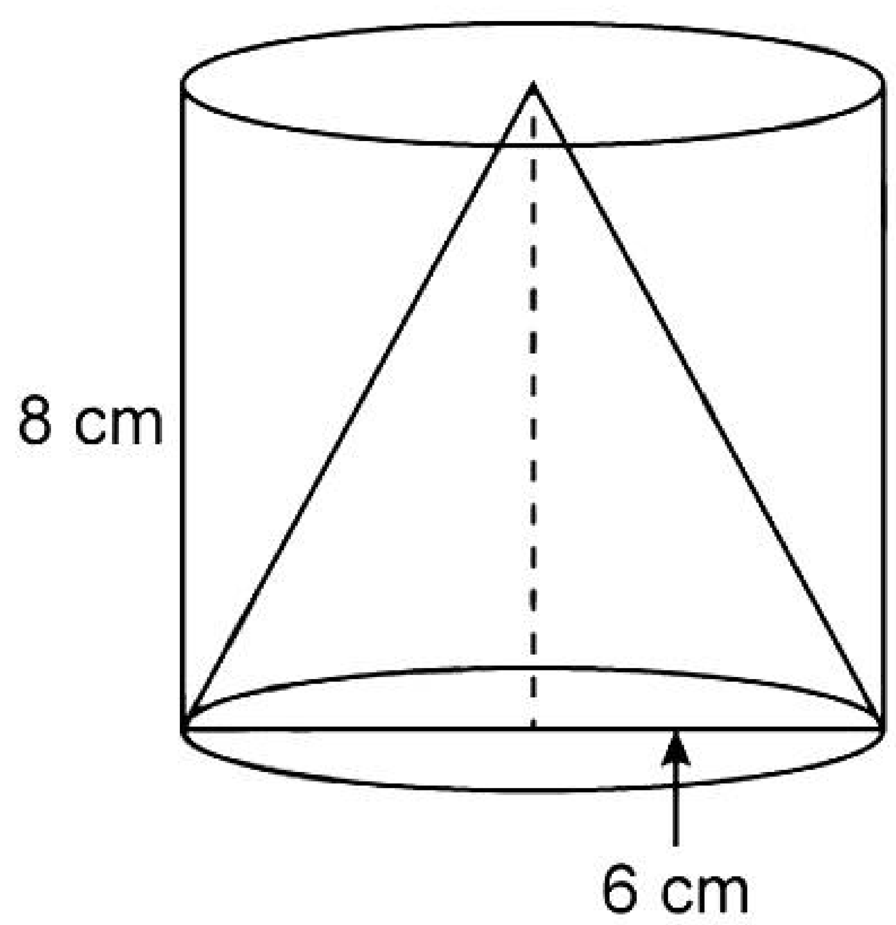
So, the required probability = $\frac{87}{360}$

19. Correct Option: D

Explanation:

Radius of the cylinder = 6 cm

Height of the cylinder = 8 cm



Volume of the cylinder

$= \pi r^2 h$ cu. units

$= \pi \times 6 \times 6 \times 8 \text{ cm}^3$

$= 288\pi \text{ cm}^3$

Volume of the cone removed

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8\ cm^3$

$= 96\pi\ cm^3$

Volume of the remaining solid = Volume of the cylinder − Volume of the cone removed

Hence, reason (R) is true.

$= 288\pi - 96\pi$

$= 192\pi\ cm^3$

Hence, assertion (A) is false, but reason (R) is true.

20. Correct Option: A

Explanation:

$\frac{4}{5}$, a, 2 are in A.P.

We know that, if p, q and r are in A.P then q − p = r − q.

So, the reason is true.

$\therefore\ a - \frac{4}{5} = 2 - a$

$\Rightarrow 2a = 2 + \frac{4}{5} = \frac{14}{5}$

$\Rightarrow a = \frac{7}{5}$

Hence, the assertion is true and reason is the correct explanation of assertion.

Section B

21. Let us assume on the contrary that $3 + 2\sqrt{5}$ is rational.

Then there exists co-prime positive integers a and b such that

$$3 + 2\sqrt{5} = \frac{a}{b}$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$\sqrt{5} = \frac{a - 3b}{2b}$$

Since, and b are integers ⇒ a – 3b is an integer.

$\Rightarrow \frac{a - 3b}{2b}$ is a rational number.

$\Rightarrow \sqrt{5}$ is rational.

This is a contradiction since $\sqrt{5}$ is irrational.

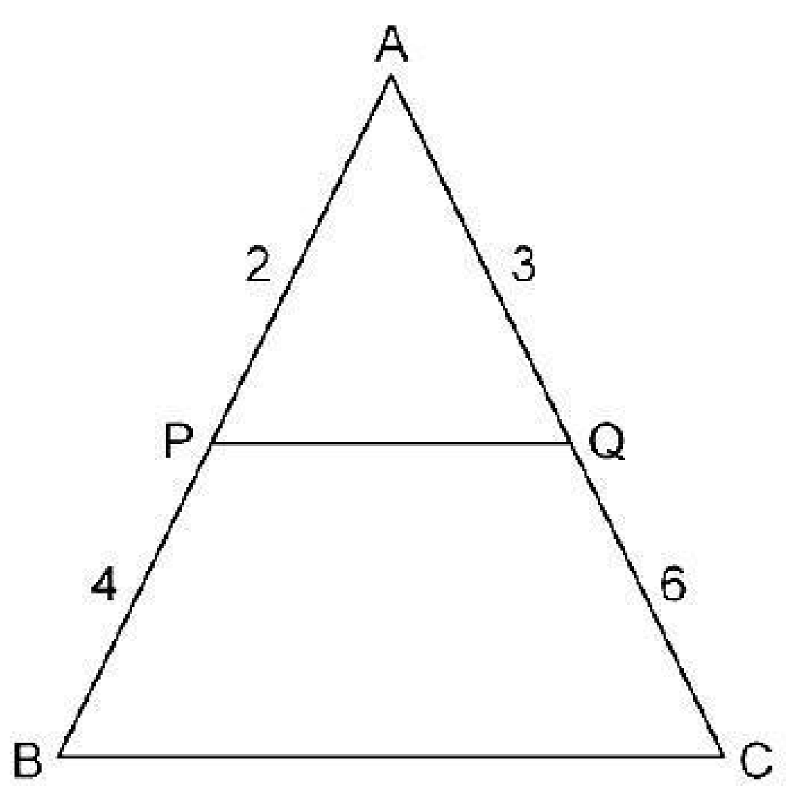
Hence, $3 + 2\sqrt{5}$ is irrational.

22. Given that P is a point on AB, then

AB = AP + PB = (2 + 4) cm = 6 cm

Also, Q is a point on AC, then

AC = AQ + QC = (3 + 6) cm = 9 cm



$$\therefore \quad \frac{AP}{AB} = \frac{2}{6} = \frac{1}{3}$$

$$\text{and} \quad \frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{AP}{AB} = \frac{AQ}{AC}$$

Thus, in $\triangle APQ$ and $\triangle ABC$

$\angle A = \angle A$ (common)

And $\frac{AP}{AB} = \frac{AQ}{AC}$

$\therefore \Delta APQ \sim \Delta ABC$ (by SAS similarity)

$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$

$\therefore \frac{PQ}{BC} = \frac{AQ}{AC}$

$\Rightarrow \frac{PQ}{BC} = \frac{3}{9} = \frac{1}{3}$

$\Rightarrow$ BC = 3 PQ

Hence proved.

23. A circle touches the sides AB, BC, CD and DA at P, Q, R and S, respectively.
We know that the length of tangents drawn from an external point to a circle are equal.
AP = AS ... (1) Tangents from A
BP = BQ ... (2) Tangents from B
CR = CQ ... (3) Tangents from C
DR = DS ... (4) Tangents from D
Adding (1), (2), (3) and (4), we get
$\therefore$ AP + BP + CR + DR = AS + BQ + CQ + DS
$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
$\Rightarrow$ AB + CD = AD + BC
$\Rightarrow$ AD = (AB + CD) − BC = {(6 + 4) − 7} cm = 3 cm
Hence, AD = 3 cm

24. L.H.S. = (sin θ + cos θ)(tan θ + cot θ)

$$= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= (\sin\theta + \cos\theta)\left(\frac{1}{\sin\theta\cos\theta}\right)$$

$$= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$$

$$= \sec\theta + \operatorname{cosec}\theta$$

= R.H.S.

OR

$$\text{L.H.S} = \frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$$
$$= \frac{1+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$$
$$= \frac{(\sec^2\theta-\tan^2\theta)+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$$
$$= \frac{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$$
$$= \frac{(\sec\theta-\tan\theta)[\sec\theta+\tan\theta+1]}{1+\sec\theta+\tan\theta}$$
$$= \sec\theta-\tan\theta$$
$$= \frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}$$
$$= \frac{1-\sin\theta}{\cos\theta}$$
$$= \text{R.H.S.}$$

25. Radius of outer circle, $r_1 = 23$ cm
Radius of inner circle, $r_2 = 12$ cm
Then,

Area of outer circle= $\pi r_1^2 = \left(\frac{22}{7}\times 23\times 23\right)\text{cm}^2 = 1662.6\ \text{cm}^2$

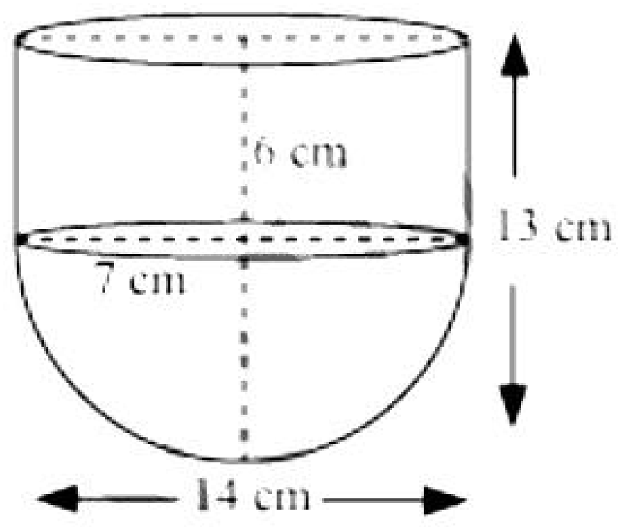
Area of inner circle= $\pi r_2^2 = \left(\frac{22}{7}\times 12\times 12\right)\text{cm}^2 = 452.6\ \text{cm}^2$

Area of ring = Area of outer circle – Area of inner circle

$= (1662.6 - 452.6)\ \text{cm}^2$

$= 1210\ \text{cm}^2$

OR



Radius (r) of cylindrical part and hemispherical part =7 cm
Height of hemispherical part = radius = 7 cm.
Height of cylindrical part (h) = 13 – 7 = 6 cm
Inner SA of the vessel = CSA of cylindrical part + CSA of hemispherical part

$= 2\pi rh + 2\pi r^2$

$= 2\times\frac{22}{7}\times 7\times 6 + 2\times\frac{22}{7}\times 7\times 7$

$= 44(6 + 7)$

$= 44 \times 13$

$= 572\ \text{cm}^2$

Section C

26. To find the minimum number of rooms required, first find the maximum number of participants which can be accommodated in each room such that the number of participants in each room is the same.
This can be determined by finding the HCF of 60, 84 and 108.

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

$\text{H.C.F.} = 2^2 \times 3 = 12$

So, the minimum number of rooms required

$= \dfrac{\text{Total number of participants}}{12}$

$= \dfrac{60 + 84 + 108}{12}$

$= 21$

27. $4s^2 - 4s + 1 = 0$

$\Rightarrow 2s - 1 = 0$

$\Rightarrow s = \dfrac{1}{2}$

So, the zeroes of $4s^2 - 4s + 1$ are $\dfrac{1}{2}$ and $\dfrac{1}{2}$.

Sum of zeroes $= \dfrac{1}{2} + \dfrac{1}{2} = 1 = \dfrac{-(-4)}{4} = \dfrac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

Product of zeroes $= \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4} = \dfrac{\text{Constant term}}{\text{Coefficient of } s^2}$

28. Let the speed of train be x km/h.

Time taken to travel 480 km = $\dfrac{480}{x}$ hrs

In second condition, let the speed of train = $(x - 8)$ km/h

It is also given that the train will take 3 more hours to cover the same distance.

Therefore, time taken to travel 480 km = $\left(\dfrac{480}{x} + 3\right)$ hrs

Speed × Time = Distance

$(x - 8)\left(\dfrac{480}{x} + 3\right) = 480$

$\Rightarrow 480 + 3x - \dfrac{3840}{x} - 24 = 480$

$\Rightarrow 3x - \dfrac{3840}{x} = 24$

$\Rightarrow 3x^2 - 24x - 3840 = 0$

$\Rightarrow x^2 - 8x - 1280 = 0$

$\Rightarrow x^2 - 40x + 32x - 1280 = 0$

$\Rightarrow x(x - 40) + 32(x - 40) = 0$

$\Rightarrow (x - 40)(x + 32) = 0$

Hence, x = 40 or -32

Since speed can't be negative, we have x = 40.

Thus speed of the train is 40 km/hr.

OR

In a cyclic quadrilateral ABCD,

$\angle A = (x + y + 10)^\circ$, $\angle B = (y + 20)^\circ$, $\angle C = (x + y - 30)^\circ$, $\angle D = (x + y)^\circ$

Then, $\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$

Now, $\angle A + \angle C = (x + y + 10)^\circ + (x + y - 30)^\circ = 180^\circ$

$\Rightarrow 2x + 2y - 20^\circ = 180^\circ$

$\Rightarrow x + y = 100$ ....(1)

And, $\angle B + \angle D = (y + 20)^\circ + (x + y)^\circ = 180^\circ$

$\Rightarrow x + 2y + 20^\circ = 180^\circ$

$\Rightarrow x + 2y = 160^\circ$ ....(2)

Subtracting (1) from (2), we get

y = 160 − 100 = 60

and x = 100 − y = 100 − 60 = 40

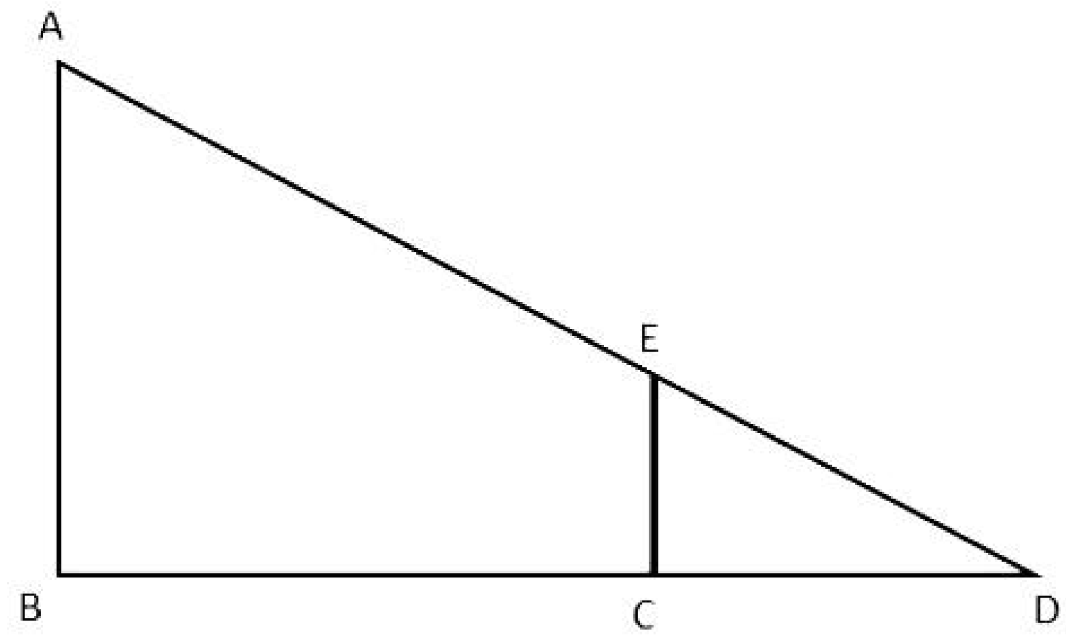
$\angle A = (x + y + 10)^\circ = (100 + 10)^\circ = 110^\circ$

$\angle B = (y + 20)^\circ = (60 + 20)^\circ = 80^\circ$

$\angle C = (x + y - 30)^\circ = (100 - 30)^\circ = 70^\circ$

$\angle D = (x + y)^\circ = 100^\circ$

29.



Let AB be the lamp post and CE be Raja's height.

Then, CE = 90 cm = 0.9 m

BC is the distance covered in 4 sec.

Hence, BC = 4×2 = 8 m

Now, CD is the length of the shadow.

In △ABD and △ECD

∠ABD = ∠ECD (each 90 degree)

∠ADB = ∠EDC (common angle)

Hence, △ABD ~ △ECD … (AA test)

$\therefore \frac{AB}{EC} = \frac{3}{0.9}$

$\therefore \frac{BD}{CD} = \frac{10}{3}$

$\therefore \frac{8 + CD}{CD} = \frac{10}{3}$

$\therefore \frac{24 + 3CD}{10} = CD$

$\therefore 2.4 + 0.3(CD) = CD$

$\therefore 2.4 = 0.7(CD)$

$\therefore x = 3.43m$

Hence, the length of Raja's shadow after 4 seconds will be 3.43 m.

OR

Given, AO = 6 m, OB = 4 m, AB = 8 m, OD = 2 m and OC = 3 m.

In $\triangle AOB$ and $\triangle COD$,

$\frac{AO}{CO} = \frac{6}{3} = 2$ and $\frac{OB}{OD} = \frac{4}{2} = 2$

$\angle AOB = \angle COD$ (vertically opposite angles)

$\therefore \triangle AOB \sim \triangle COD$... (SAS test)

$\therefore \frac{AB}{CD} = 2$ (Corresponding angles of similar triangles)

$\therefore CD = \frac{1}{2} AB = 4$ m

Hence, the length of CD is 4 m.

30. In $\triangle PQR$, by applying Pythagoras theorem

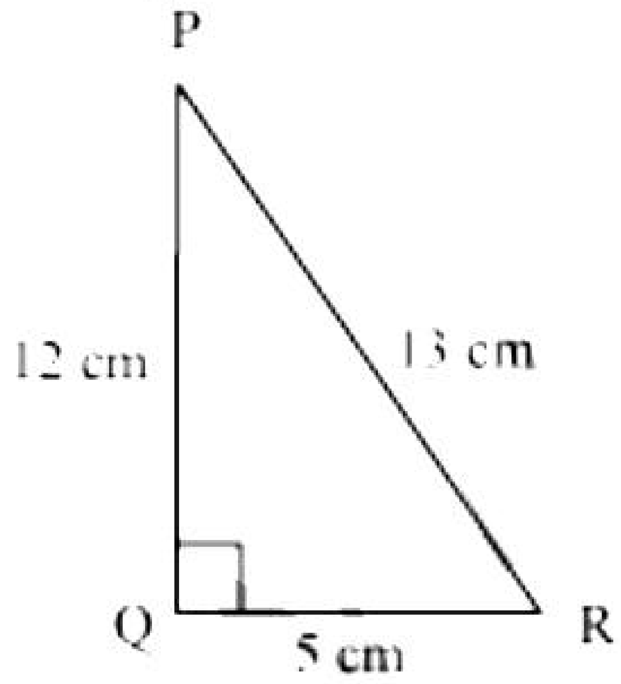
$PR^2 = PQ^2 + QR^2$

$(13)^2 = (12)^2 + QR^2$

$169 = 144 + QR^2$

$25 = QR^2$

QR = 5 cm



$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{5}{12}$

$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ} = \frac{5}{12}$

$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

31. Two dice are thrown simultaneously.
Total number of outcomes = $6 \times 6 = 36$

i. 5 will not come up on either of them.
Favorable cases are
(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 1), (2, 2),
(2, 3), (2, 4), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4),
(3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 6), (6, 1),
(6, 2), (6, 3), (6, 4), (6, 6) = 25

$\therefore$ Probability that 5 will not come up on either die $= \frac{25}{36}$

ii. 5 will not come up on at least one.
Favorable cases are
(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1),
(5, 2), (5, 3), (5, 4), (5, 6) = 11

Probability that 5 will come at least once $= \frac{11}{36}$

iii. 5 will come up on both dice.
Favourable case: (5, 5)

$\therefore$ Probability that 5 will come on both dice $= \frac{1}{36}$

Section D

32. Let the present age of Rehman be x years.
Three years ago, his age was (x − 3) years.
Five years hence, his age will be (x + 5) years.
It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\therefore \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$
$$\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$
$$\frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$$
$$\Rightarrow 3(2x+2)=(x-3)(x+5)$$
$$\Rightarrow 6x+6=x^2+2x-15$$
$$\Rightarrow x^2-4x-21=0$$
$$\Rightarrow x^2-7x+3x-21=0$$
$$\Rightarrow x(x-7)+3(x-7)=0$$
$$\Rightarrow (x-7)(x+3)=0$$
$$\Rightarrow x=7,-3$$

However, age cannot be negative.
Therefore, Rehman's present age is 7 years.

OR

Let the larger and smaller numbers be x and y respectively. According to the given question,

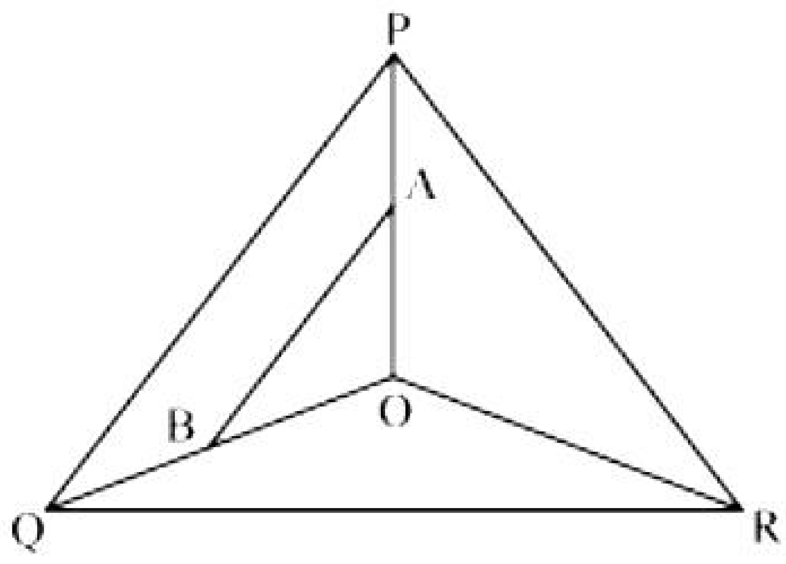
$$x^2-y^2=180 \text{ and } y^2=8x$$
$$\Rightarrow x^2-8x=180$$
$$\Rightarrow x^2-8x-180=0$$
$$\Rightarrow x^2-18x+10x-180=0$$
$$\Rightarrow x(x-18)+10(x-18)=0$$
$$\Rightarrow (x-18)(x+10)=0$$
$$\Rightarrow x=18,-10$$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.
Therefore, the larger number will be 18 only.

$$x=18$$
$$\therefore\ y^2=8x=8\times 18=144$$
$$\Rightarrow\ y=\pm\sqrt{144}=\pm 12$$
$$\therefore\ \text{Smaller number}=\pm 12$$

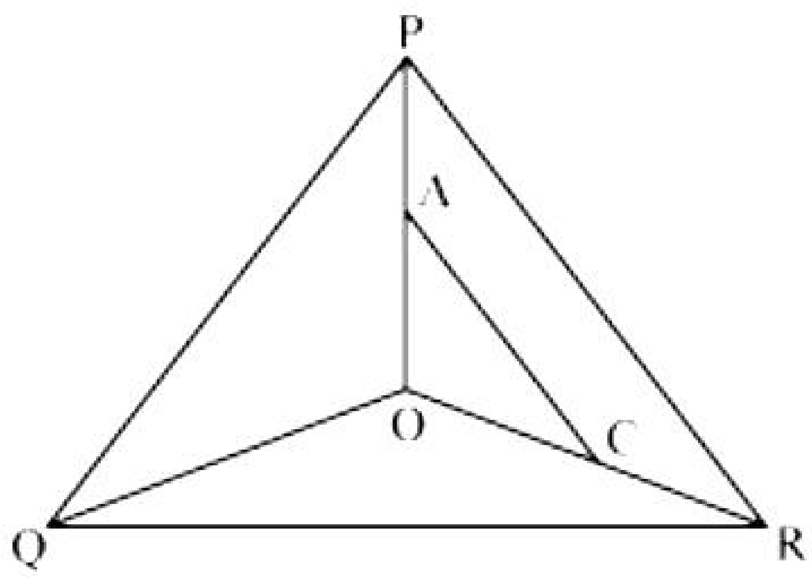
Therefore, the numbers are 18 and 12 or 18 and −12.

33.



In ΔPOQ, AB || PQ,

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ (i) [By basic proportionality theorem]



In ΔPOR, AC || PR,

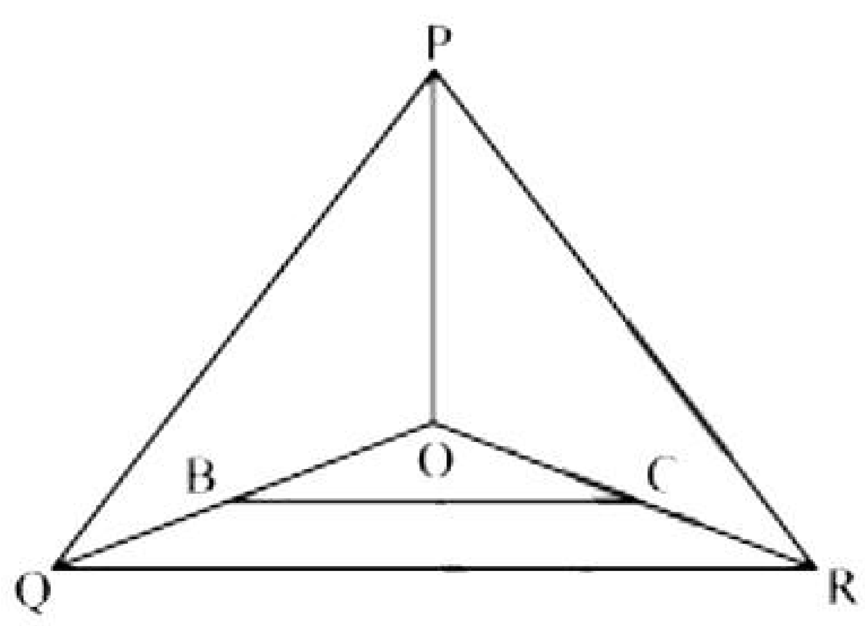
$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ (ii) [By basic proportionality theorem]

From (i) and (ii)

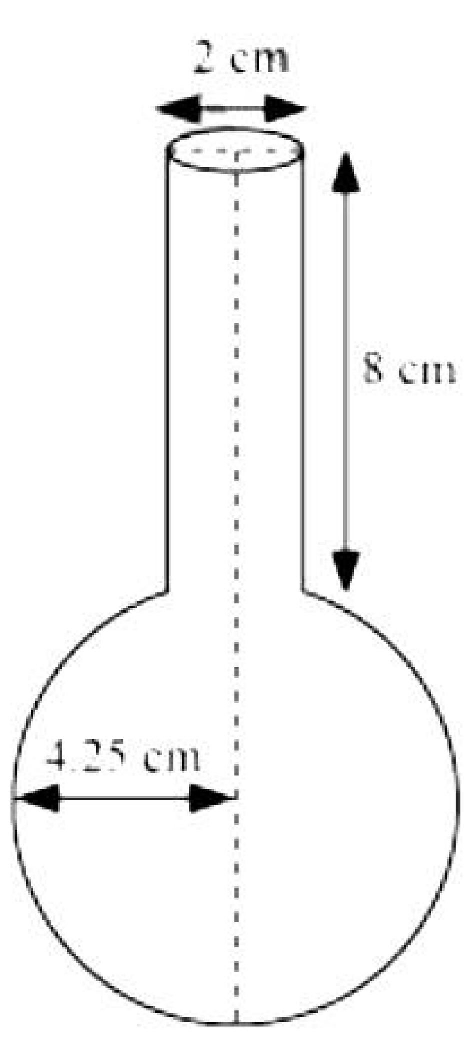
$\frac{OB}{BQ} = \frac{OC}{CR}$

Therefore BC || QR

(By converse of basic proportionality theorem)



34.



Radius (r_1) of spherical part = 8.5/2

Height (h) of cylindrical part = 8 cm

Radius (r_2) of cylindrical part = $\frac{2}{2} = 1$ cm

Volume of vessel = volume of sphere + volume of cylinder

$$= \frac{4}{3}\pi r_1^3 + \pi r_2^2 h$$

$$= \frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3 + \pi(1)^2(8)$$

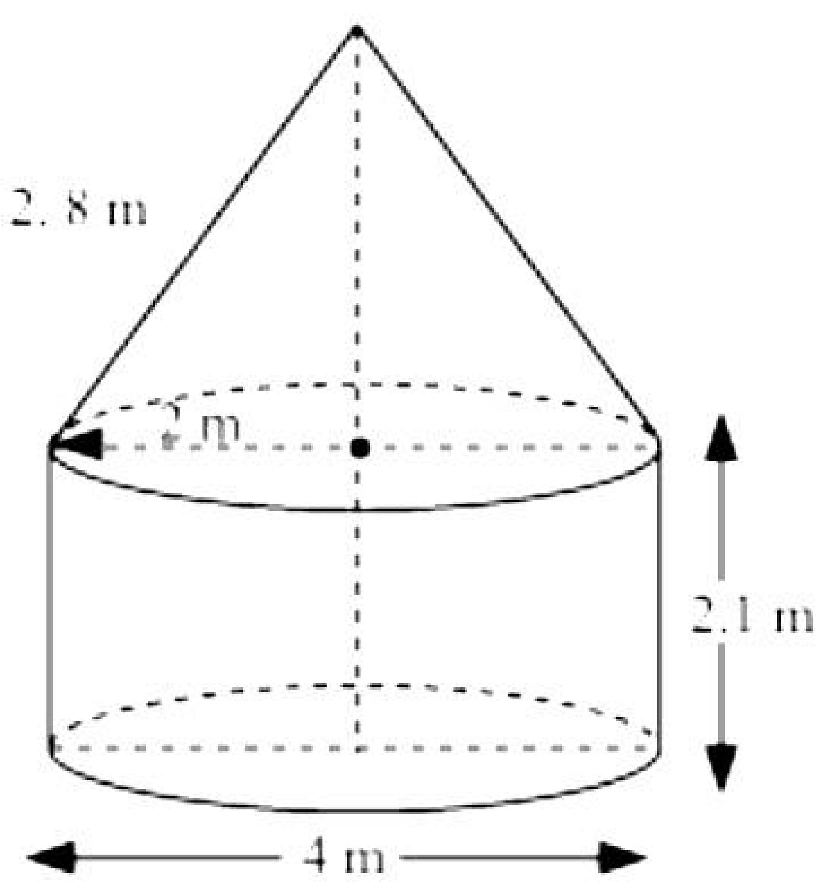
$$= \frac{4}{3} \times 3.14 \times \left(\frac{8.5}{2}\right)^3 + 3.14 \times 8$$

$$= 321.39 + 25.12$$

$$= 346.51 \text{ cm}^3$$

Hence, she is wrong.

OR



Given that

Height (h) of the cylindrical part = 2.1 m

Diameter of the cylindrical part = 4 m

So, radius (r) of the cylindrical part = 2 m

Slant height (l) of conical part = 2.8 m

Area of canvas used = CSA of conical part + CSA of cylindrical part

$$= \pi rl + 2\pi rh$$

$$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$$

$$= 2\pi[2.8 + 4.2]$$

$$= 2 \times \frac{22}{7} \times 7$$

$$= 44 \text{ m}^2$$

Cost of 1 m^2 canvas = Rs. 500

Cost of 44 m^2 canvas = Rs. (44 × 500) = Rs. 22000

So, it will cost Rs. 22000 for making such tent.

35. We may find class mark of each interval by using the relation

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Now taking 17 as assumed mean (a) we may calculate d_i and f_id_i as following–

Number of days	**Number of students f_i**	x_i	$d_i = x_i - 17$	f_id_i
0 – 6	11	3	−14	−154
6 – 10	10	8	−9	−90
10 – 14	7	12	−5	−35
14 – 20	4	17	0	0
20 – 28	4	24	7	28
28 – 38	3	33	16	48
38 – 40	1	39	22	22
Total	40			−181

Now we may observe that

$\sum f_i = 40$

$\sum f_id_i = -181$

$$\text{mean} \quad \bar{x} = a + \left(\frac{\sum f_id_i}{\sum f_i}\right)$$

$$= 17 + \left(\frac{-181}{40}\right)$$

$$= 17 - 4.525$$

$$= 12.475$$

$$= 12.48$$

So, mean number of days is 12.48 days, for which a student was absent.

Section E

36.

i. Here, the chocolates are arranged in increasing order of 2.
Thus, it forms an A.P. with a = 3 and d = 2.
Therefore, the required A.P. is 3, 5, 7, …..
Given, $S_n = 120$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow 120 = \frac{n}{2}\left[2 \times 3 + (n-1)2\right]$$

$$\Rightarrow 240 = \left(6n + 2n^2 - 2n\right)$$

$$\Rightarrow n^2 + 2n - 120 = 0$$

$$\Rightarrow (n+12)(n-10) = 0$$

$$\Rightarrow (n+12) = 0 \text{ or } (n-10) = 0$$

$$\Rightarrow n = -12 \text{ or } n = 10$$

Number of rows can't be negative.
Hence, total number of rows of chocolates is 10.

ii. Here, a = 3, d = 2 and n = 10
$a_n = a_{10} = a + (n - 1)d = 3 + (10 - 1)2 = 21$
Hence, 21 chocolates are placed in last row.

OR

We have, d = 2 and $a_n = a + (n - 1)d$
$\Rightarrow a_7 - a_3 = a + 6d - a - 2d = 4d = 4(2) = 8$
Hence, the difference in number of chocolates placed in 7^{th} and 3^{rd} row is 8.

iii. Here, n = 15

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow S_{15} = \frac{15}{2}\left[2 \times 3 + 14 \times 2\right] = \frac{15 \times 34}{2} = 255$$

Hence, 255 chocolates will be placed by her with the same arrangement.

37.

i. Distance covered by Bus No. 735 = OA

$A(4,4)$ and $O(0,0)$.

$AO = \sqrt{(0-4)^2 + (0-4)^2} = 4\sqrt{2}$ km

ii. Distance between locations B and A = AB

$A(4,4)$ and $B(3,1)$.

$AB = \sqrt{(4-3)^2 + (4-1)^2} = \sqrt{10}$ km

OR

Distance between locations O and B.

$B(3,1)$ and $O(0,0)$

$OB = \sqrt{(0-3)^2 + (0-1)^2} = \sqrt{10}$ km

iii. distance covered by Bus No. 736 = O – B – A

$A(4,4)$, $B(3,1)$ and $O(0,0)$.

$OB = \sqrt{(0-3)^2 + (0-1)^2} = \sqrt{10}$ km

$AB = \sqrt{(4-3)^2 + (4-1)^2} = \sqrt{10}$ km

$O - B - A = 2\sqrt{10}$ km

38.

i. Height of the pole is 10 m which is AX.

AB = AX – XB = 10 – 4 = 6 m

ii. In right-angled ΔBAC,

$\tan 60^\circ = \frac{AB}{AC}$

$\Rightarrow \sqrt{3} = \frac{6}{AC}$

$\Rightarrow AC = 2\sqrt{3}$ m

Hence, the distance between foot of the ladder and the pole is $2\sqrt{3}$ m.

OR

In right-angled ΔBAC,

$\sin 60^\circ = \frac{AB}{BC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{6}{BC}$

$\Rightarrow BC = 4\sqrt{3}$ m

iii. If $AB = AC \Rightarrow \angle ACB = \angle ABC$

And, $\angle BAC = 90^\circ$

Then, in ΔABC,

$\angle ACB + \angle ABC + \angle BAC = 180^\circ$

$\Rightarrow 2\angle ACB + 90^\circ = 180^\circ$

$\Rightarrow 2\angle ACB = 90^\circ$

$\Rightarrow \angle ACB = 45^\circ$

Thus, the angle made by the ladder with the ground must be 45°.